3.1 Newton's Law of Universal Gravitation

SECTION EXPECTATIONS

- Describe Newton's law of universal gravitation.
- Apply Newton's law of universal gravitation quantitatively.

KEY TERMS

- Tychonic system
- Kepler's laws
- law of universal gravitation

In previous science courses, you learned about the Ptolemaic system for describing the motion of the planets and the Sun. The system developed by Ptolemy (151–127 B.C.E.) was very complex because it was geocentric, that is, it placed Earth at the centre of the universe. In 1543, Nicholas Copernicus (1473–1543) proposed a much simpler, heliocentric system for the universe in which Earth and all of the other planets revolved around the Sun. The Copernican system was rejected by the clergy, however, because the religious belief system at the time placed great importance on humans and Earth as being central to a physically perfect universe. You probably remember learning that the clergy put Galileo Galilei (1564–1642) on trial for supporting the Copernican system.

Have you ever heard of the **Tychonic system**? A famous Danish nobleman and astronomer, Tycho Brahe (1546–1601), proposed a system, shown in Figure 3.1, that was intermediate between the Ptolemaic and Copernican systems. In Brahe's system, Earth is still and is the centre of the universe; the Sun and Moon revolve around Earth, but the other planets revolve around the Sun. Brahe's system captured the interest of many scientists, but never assumed the prominence of either the Ptolemaic or Copernican systems. Nevertheless, Tycho Brahe contributed a vast amount of detailed, accurate information to the field of astronomy.

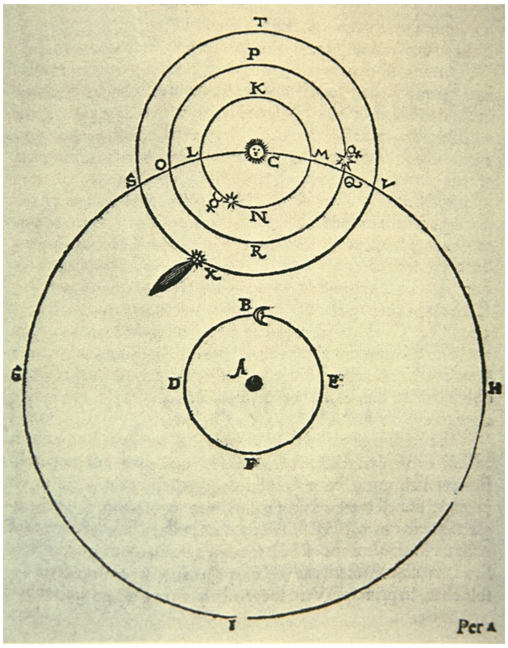


Figure 3.1 The Tychonic universe was acceptable to the clergy, because it maintained that Earth was the centre of the universe. The system was somewhat satisfying for scientists, because it was simpler than the Ptolemaic system.

Laying the Groundwork for Newton

Astronomy began to come of age as an exact science with the detailed and accurate observations of Tycho Brahe. For more than 20 years, Brahe kept detailed records of the positions of the planets and stars. He catalogued more than 777 stars and, in 1572, discovered a new star that he named "Nova." Brahe's star was one of very few supernovae ever found in the Milky Way galaxy.

In 1577, Brahe discovered a comet and demonstrated that it was not an atmospheric phenomenon as some scientists had believed, but rather that its orbit lay beyond the Moon. In addition to making observations and collecting data, Brahe designed and built the most accurate astronomical instruments of the day (see Figure 3.2). In addition, he was the first astronomer to make corrections for the refraction of light by the atmosphere.

HISTORY LINK

Tycho Brahe was a brilliant astronomer who led an unusual and tumultuous life. At age 19, he was involved in a duel with another student and part of his nose was cut off. For the rest of his life, Brahe wore an artificial metal nose.

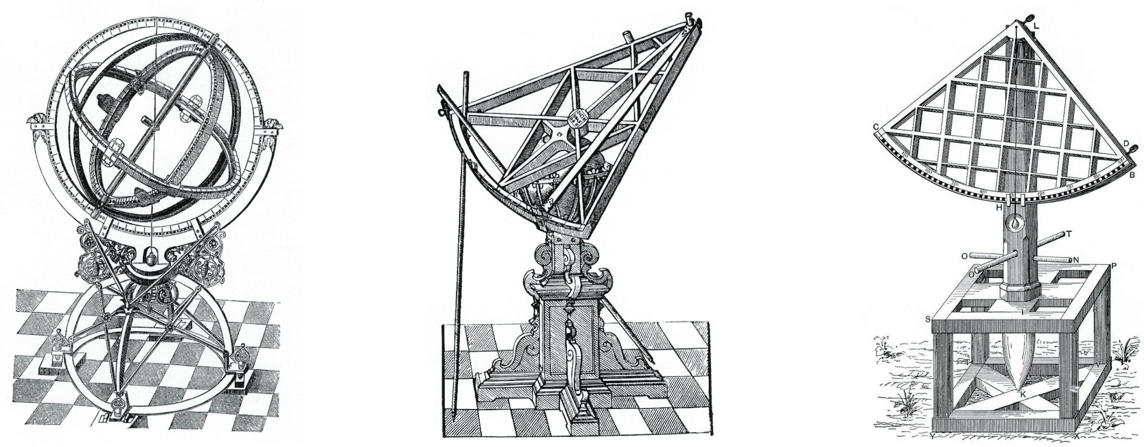


Figure 3.2 Brahe's observatory in Hveen, Denmark, contained gigantic instruments that, without magnification, were precise to 1/30 of a degree.

In 1600, Brahe invited Kepler to be one of his assistants. Brahe died suddenly the following year, leaving all of his detailed data to Kepler. With this wealth of astronomical data and his ability to perform meticulous mathematical analyses, Kepler discovered three empirical relationships that describe the motion of the planets. These relationships are known today as **Kepler's laws**.

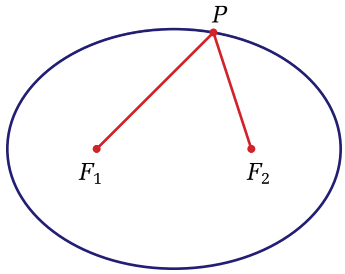
KEPLER'S LAWS

1. Planets move in elliptical orbits, with the Sun at one focus of the ellipse.
2. An imaginary line between the Sun and a planet sweeps out equal areas in equal time intervals.
3. The quotient of the square of the period of a planet's revolution around the Sun and the cube of the average distance from the Sun is constant and the same for all planets.

$$\frac{T^2}{r^3} = k \quad \text{or} \quad \frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3},$$ where A and B are two planets.

MATH LINK

A circle is a special case of an ellipse. An ellipse is defined by two focuses and the relationship $\overline{F_1P} + \overline{F_2P} = k$, where k is a constant and is the same for every point on the ellipse. If the two focuses of an ellipse are brought closer and closer together until they are superimposed on each other, the ellipse becomes a circle.



Kepler's first law does not sound terribly profound, but he was contending not only with scientific observations of the day, but also with religious and philosophical views. For centuries, the perfection of "celestial spheres" was of extreme importance in religious beliefs. Ellipses were not considered to be "perfect," so many astronomers resisted accepting any orbit other than a "perfect" circle that fit on the surface of a sphere. However, since Kepler published his laws, there has never been a case in which the data for the movement of a satellite, either natural or artificial, did not fit an ellipse.

Kepler's second law is illustrated in Figure 3.3. Each of the shaded sections of the ellipse has an equal area. According to Kepler's second law, therefore, the planet moves along the arc of each section in the same period of time. Since the arcs close to the Sun are longer than the arcs more distant from the Sun, the planet must be moving more rapidly when it is close to the Sun.

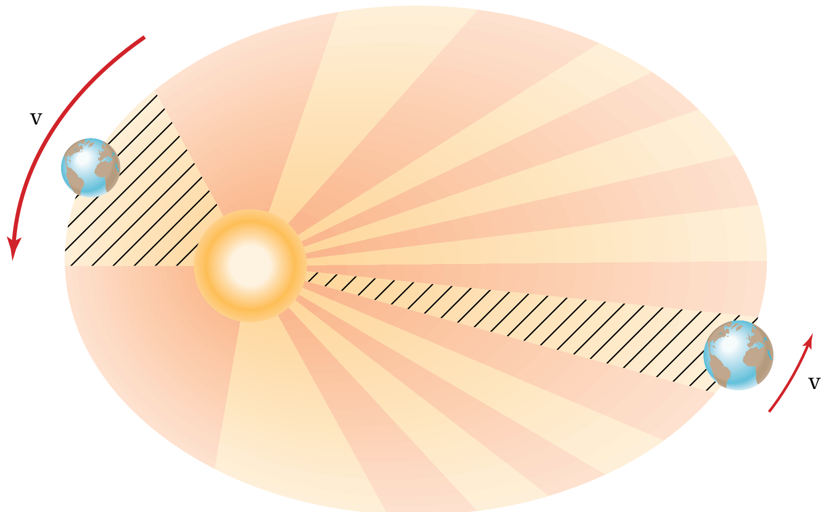


Figure 3.3 According to Kepler's second law, the same length of time was required for a planet to move along each of the arcs at the ends of the segments of the ellipse. Kepler could not explain why planets moved faster when they were close to the Sun than when they were farther away.

When Kepler published his third law, he had no way of knowing the significance of the constant in the mathematical expression $T^2/r^3 = k$. All he knew was that the data fit the equation. Kepler suspected that the Sun was in some way influencing the motion of the planets, but he did not know how or why this would lead to the mathematical relationship. The numerical value of the constant in Kepler's third law and its relationship to the interaction between the Sun and the planets would take on significance only when Sir Isaac Newton (1642–1727) presented his law of universal gravitation.

Universal Gravitation

Typically in research, the scientist makes some observations that lead to an hypothesis. The scientist then tests the hypothesis by planning experiments, accumulating data, and then comparing the results to the hypothesis. The development of Newton's law of universal gravitation happened in reverse. Brahe's data and Kepler's analysis of the data were ready and waiting for Newton to use to test his hypothesis about gravity.

Newton was not the only scientist of his time who was searching for an explanation for the motion, or orbital dynamics, of the planets. In fact, several scientists were racing to see who could find the correct explanation first. One of those scientists was astronomer Edmond Halley (1656–1742). Halley and others, based on their calculations, had proposed that the force between the planets and the Sun decreased with the square of the distance between a planet and the Sun. However, they did not know how to apply that concept to predict the shape of an orbit.

Halley decided to put the question to Newton. Halley first met Newton in 1684, when he visited Cambridge. He asked Newton what type of path a planet would take if the force attracting it to the Sun decreased with the square of the distance from the Sun. Newton quickly answered, "An elliptical path." When Halley asked him how he knew, Newton replied that he had made that calculation many years ago, but he did not know where his calculations were. Halley urged Newton to repeat the calculations and send them to him.

Three months later, Halley's urging paid off. He received an article from Newton entitled "De Motu" ("On Motion"). Newton continued to improve and expand his article and in less than three years, he produced one of the most famous and fundamental scientific works: *Philosophiae Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Philosophy*). The treatise contained not only the law of universal gravitation, but also Newton's three laws of motion.

Possibly, Newton was successful in finding the law of universal gravitation because he extended the concept beyond the motion of planets and applied it to all masses in all situations. While other scientists were looking at the motion of planets, Newton was watching an apple fall from a tree to the ground. He reasoned that the same attractive force that existed between the Sun and Earth was also responsible for attracting the apple to Earth. He also reasoned that the force of gravity acting on a falling object was proportional to the mass of the object. Then, using his own third law of action-reaction forces, if a falling object such as an apple was attracted to Earth, then Earth must also be attracted to the apple, so the force of gravity must also be proportional to the mass of Earth. Newton therefore proposed that *the force of gravity between any two objects is proportional to the product of their*

HISTORY LINK

Sir Edmond Halley, the astronomer who prompted Newton to publish his work on gravitation, is the same astronomer who discovered the comet that was named in his honour — Halley's Comet. Without Halley's urging, Newton might never have published his famous *Principia*, greatly slowing the progress of physics.

masses and inversely proportional to the square of the distance between their centres — the **law of universal gravitation**. The mathematical equation for the law of universal gravitation is given in the following box.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

The force of gravity is proportional to the product of the two masses that are interacting and inversely proportional to the square of the distance between their centres.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Quantity	Symbol	SI unit
force of gravity	F_g	N (newtons)
first mass	m_1	kg (kilograms)
second mass	m_2	kg (kilograms)
distance between the centres of the masses	r	m (metres)
universal gravitational constant	G	$\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{kg}^2}$ (newton · metre squared per kilogram squared)

Unit Analysis

$$\text{newton} = \left(\frac{\text{newton}\cdot\ \text{metre}^2}{\text{kilogram}^2}\right)\left(\frac{\text{kilogram}\cdot\text{kilogram}}{\text{metre}^2}\right)$$

$$\left(\frac{\mathrm{N}\cdot\cancel{\mathrm{m}^2}}{\cancel{\mathrm{kg}^2}}\right)\left(\frac{\cancel{\mathrm{kg}}\cdot\cancel{\mathrm{kg}}}{\cancel{\mathrm{m}^2}}\right) = \mathrm{N}$$

Note: The value of the universal gravitational constant is

$$G = 6.67 \times 10^{-11}\ \frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{kg}^2}.$$

Conceptual Problem

- You have used the equation $F_g = mg$ many times to calculate the weight of an object on Earth's surface. Now, you have learned that the weight of an object on Earth's surface is $F_g = G\frac{m_E m_o}{r_{E\text{-}o}^2}$, where m_E is the mass of Earth, m_o is the mass of the object, and $r_{E\text{-}o}$ is the distance between the centres of Earth and the object. Explain how the two equations are related. Express g in terms of the variables and constant in Newton's law of universal gravitation.

SAMPLE PROBLEM

Weighing an Astronaut

A 65.0 kg astronaut is walking on the surface of the Moon, which has a mean radius of 1.74×10^3 km and a mass of 7.35×10^{22} kg. What is the weight of the astronaut?

Conceptualize the Problem

- The weight of the astronaut is the gravitational force on her.
- The relationship $F_g = mg$, where $g = 9.81 \frac{m}{s^2}$, *cannot* be used in this problem, since the astronaut is not on Earth's surface.
- The law of universal gravitation applies to this problem.

Identify the Goal

The gravitational force, F_g, on the astronaut

Identify the Variables and Constants

Known	Implied	Unknown
$m_M = 7.35 \times 10^{22}$ kg	$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	F_g
$m_a = 65.0$ kg		
$r = 1.74 \times 10^3$ km (1.74×10^6 m)		

Develop a Strategy

Apply the law of universal gravitation.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Substitute the numerical values and solve.

$$F_g = \left(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right)\frac{(7.35 \times 10^{22}\ kg)(65.0\ kg)}{(1.74 \times 10^6\ m)^2}$$

$$F_g = 105.25\ N$$

$$F_g \cong 105\ N$$

The weight of the astronaut is approximately 105 N.

Validate the Solution

Weight on the Moon is known to be much less than that on Earth. The astronaut's weight on the Moon is about one sixth of her weight on Earth ($65.0\ kg \times 9.81 \frac{m}{s^2} \cong 638\ N$), which is consistent with this common knowledge.

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PRACTICE PROBLEMS

1. Find the gravitational force between Earth and the Sun. (See Appendix B, Physical Constants and Data.)
2. Find the gravitational force between Earth and the Moon. (See Appendix B, Physical Constants and Data.)
3. How far apart would you have to place two 7.0 kg bowling balls so that the force of gravity between them would be 1.25×10^{-4} N? Would it be possible to place them at this distance? Why or why not?
4. Find the gravitational force between the electron and the proton in a hydrogen atom if they are 5.30×10^{-11} m apart. (See Appendix B, Physical Constants and Data.)
5. On Venus, a person with mass 68 kg would weigh 572 N. Find the mass of Venus from this data, given that the planet's radius is 6.31×10^{6} m.
6. In an experiment, an 8.0 kg lead sphere is brought close to a 1.5 kg mass. The gravitational force between the two objects is 1.28×10^{-8} N. How far apart are the centres of the objects?
7. The radius of the planet Uranus is 4.3 times the radius of earth. The mass of Uranus is 14.7 times Earth's mass. How does the gravitational force on Uranus' surface compare to that on Earth's surface?
8. Along a line connecting Earth and the Moon, at what distance from Earth's centre would an object have to be located so that the gravitational attractive force of Earth on the object was equal in magnitude and opposite in direction from the gravitational attractive force of the Moon on the object?

Gravity and Kepler's Laws

The numerical value of *G*, the universal gravitational constant, was not determined experimentally until more than 70 years after Newton's death. Nevertheless, Newton could work with concepts and proportionalities to verify his law.

Newton had already shown that the inverse square relationship between gravitational force and the distance between masses was supported by Kepler's first law — that planets follow elliptical paths.

Kepler's second law showed that planets move more rapidly when they are close to the Sun and more slowly when they are farther from the Sun. The mathematics of elliptical orbits in combination with an inverse square relationship to yield the speed of the planets is somewhat complex. However, you can test the concepts graphically by completing the following investigation.